10.1.1 Simplifying Numerical Square Root Expressions

Definitions

1. The square of an integer is called a **perfect square integer**.
   Since $1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16$, etc. . . , the perfect square integers are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225 . . .

2. The **square root of a number *x*** is the number whose square is *x*.

   Notation: $\sqrt{x}$ is read "the square root of *x*"

   The $\sqrt{}$ symbol is called the **radical symbol**. The argument inside radical symbol, the *x*, is called the **radicand**.

e.g.

$\sqrt{1} =$ ____	is asking	$(\quad)^2 = 1$
$\sqrt{4} =$ ____	is asking	$(\quad)^2 = 4$
$\sqrt{9} =$ ____	is asking	$(\quad)^2 = 9$
$\sqrt{16} =$ ____	is asking	$(\quad)^2 = 16$
$\sqrt{25} =$ ____	is asking	$(\quad)^2 = 25$
$\sqrt{36} =$ ____	is asking	$(\quad)^2 = 36$
$\sqrt{49} =$ ____	is asking	$(\quad)^2 = 49$
$\sqrt{64} =$ ____	is asking	$(\quad)^2 = 64$
$\sqrt{81} =$ ____	is asking	$(\quad)^2 = 81$
$\sqrt{100} =$ ____	is asking	$(\quad)^2 = 100$
$\sqrt{121} =$ ____	is asking	$(\quad)^2 = 121$
$\sqrt{144} =$ ____	is asking	$(\quad)^2 = 144$
$\sqrt{169} =$ ____	is asking	$(\quad)^2 = 169$
$\sqrt{196} =$ ____	is asking	$(\quad)^2 = 196$
$\sqrt{225} =$ ____	is asking	$(\quad)^2 = 225$

Fact
The square root of a number that is not a perfect square is an **irrational number**. In decimal form an irrational number is both non-terminating ar non-repeating. Decimal numbers that terminate or repeat are **rational numbers** and can be expressed as a ratio of two integers (a fraction).

e.g. $\sqrt{5} = 2.236067977...$
$\sqrt{11} = 3.31662479...$

Fact The square root of a negative number is not a real number.

i.e. Since $\sqrt{-9} =$ ____ is asking $(\quad)^2 = -9$, $\sqrt{-9}$ is not a real number

Product Property of Square Roots

If a and b are nonnegative real numbers, then $\sqrt{ab} = \sqrt{a}\sqrt{b}$

Definition
A **radical expression is in simplest form** when:
1. The radicand contains no perfect square factors (other than one).
2. The radicand contains no fractions.
3. The denominator of a fraction contains no radical terms.

To simplify $\sqrt{96}$
1. Factor 96 so that one of the factors is a perfect square. For the numbe 96 there are two factorizations to choose from.
$$\sqrt{96} = \sqrt{4(24)} = \sqrt{16(6)}$$
The simplification is faster when the factorization with the largest per square factor is chosen.
2. Apply the Product Property of Square Roots $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

$$\sqrt{96} = \sqrt{4(24)} = \sqrt{4}\sqrt{24} = 2\sqrt{24} = 2\sqrt{4(6)} = 2\sqrt{4}\sqrt{6} = 4\sqrt{6}$$
$$\sqrt{96} = \sqrt{16(6)} = \sqrt{16}\sqrt{6} = 4\sqrt{6}$$

Simplify

1. $3\sqrt{160}$

2. $-4\sqrt{50}$

3. $-2\sqrt{216}$

4. $5\sqrt{600}$

5. $-9\sqrt{243}$

10.1.2 Simplifying Square Root Expressions

Square Root Properties

1. For any real number a, $\sqrt{a^2} = |a|$
2. For any real numbe $a \geq 0$, $\sqrt{a^2} = a$

Example
Illustrate property 1 by letting $a = 4$ and $a = -6$.

Discovery Exercise - Consider the following.

1. $\sqrt{x^2}$ = ____ is asking what squared is x^2, that is $(\quad)^2 = x^2$
2. $\sqrt{x^4}$ = ____ is asking what squared is x^4, that is $(\quad)^2 = x^4$
3. $\sqrt{x^6}$ = ____ is asking what squared is x^6, that is $(\quad)^2 = x^6$
4. $\sqrt{x^8}$ = ____ is asking what squared is x^8, that is $(\quad)^2 = x^8$

Fact

Factors with even exponents are perfect square factors. To find the square root of a perfect square factor, remove the radical symbol and divide the exponent by two.

Examples

1. Simplify $2\sqrt{8a^2}$
2. Simplify $-4x\sqrt{48x^2y^4}$
3. Simplify $9\sqrt{242x^2}$
4. Simplify $-3d\sqrt{54c^2d^4}$
5. Simplify $-6xy^2\sqrt{72x^4y^2}$
6. Simplify $12a\sqrt{288a^6b^2}$

Recall One of the requirements for a radical expression to be in **simplest form** is that the radicand contain no perfect square factors.

To Simplify $\sqrt{x^n}$ when n is an odd integer. e.g. $\sqrt{x^5}$

1. Rewrite $x^n = x^{n-1}(x)$ e.g. $\sqrt{x^5} = \sqrt{x^4(x)}$

2. Apply $\sqrt{ab} = \sqrt{a}\sqrt{b}$ and simplify. $\sqrt{x^4(x)} = \sqrt{x^4} \cdot \sqrt{x} = x^2\sqrt{x}$

Examples

1. Simplify $\sqrt{x^7}$

2. Simplify $\sqrt{a^{19}}$

3. Simplify $\sqrt{a^3b^{11}}$

4. Simplify $\sqrt{c^9d^{10}}$

5. Simplify $\sqrt{u^{15}v^4}$

6. Simplify $\sqrt{(x-4)^4}$

7. Simplify $\sqrt{(2a+5)^7}$

To simplify the radical expression $-2a\sqrt{32a^5b^{13}}$

1. Factor the radicand into a product of perfect square factors and factor: that are not perfect squares.

2. Apply the product property of square roots $\sqrt{ab} = \sqrt{a}\sqrt{b}$ and simplif

Simplify

1. $\sqrt{27x^7}$

2. $\sqrt{45a^{11}}$

3. $3x\sqrt{18x^3y^{10}}$

4. $-2a\sqrt{72a^3b^{16}}$

5. $\sqrt{36(x+4)^4}$

6. $\sqrt{162(2x-3)^5}$

10.2 Add and Subtract Radical Expressions

Recall The Distributive Property is used to combine like terms.

e.g. $3x + 5x = (3+5)x = 8x$

similarly $3\sqrt{7} + 5\sqrt{7} = (3+5)\sqrt{7} = 8\sqrt{7}$

New Definition of Like Terms
Like terms have identical variable and radical factors. To combine like terms simply add the coefficients leaving the variable and radical factors a

Simplify

1. $5\sqrt{3} - 8\sqrt{3}$
2. $2\sqrt{5} - 3\sqrt{5}$
3. $4\sqrt{18} - 6\sqrt{2}$
4. $2\sqrt{50} + \sqrt{32}$
5. $5\sqrt{18a} - 8\sqrt{32a}$
6. $2\sqrt{12y^3} - 4y\sqrt{27y}$
7. $2x\sqrt{8y} - 4\sqrt{18x^2y} + 5\sqrt{32x^2y}$
8. $3\sqrt{27x^5} - 4x\sqrt{12x^3} + 5x^2\sqrt{75x}$
9. $5\sqrt{4c-8d} - 4\sqrt{9c-18d}$

10.3 Multiply and Divide Radical Expressions

Recall If $a \geq 0$ and $b \geq 0$, then $\sqrt{ab} = \sqrt{a}\sqrt{b}$

Multiply.

1. $\sqrt{5a}\sqrt{7b}$

2. $\sqrt{7x^2}\sqrt{7x^5}$

3. $\sqrt{2a^2}\sqrt{3a^5b}\sqrt{6ab^2}$

Fact If $a \geq 0$, then $\sqrt{a}\sqrt{a} = \left(\sqrt{a}\right)^2 = a$.

Multiply.

1. $\sqrt{x}\sqrt{x}$

2. $\sqrt{5y}\sqrt{5y}$

3, $\sqrt{a-3}\sqrt{a-3}$

4. $\sqrt{a^2-2a+5}\sqrt{a^2-2a+5}$

Multiply.

1. $\sqrt{3ab}\left(\sqrt{4a}-\sqrt{12b}\right)$

2, $\sqrt{7x}\left(\sqrt{7x}-\sqrt{14y}\right)$

3. $\left(2\sqrt{x}-3\sqrt{y}\right)\left(5\sqrt{x}-7\sqrt{y}\right)$

Recall $(a+b)(a-b)=a^2-b^2$

Multiply.

1. $\left(\sqrt{3}+\sqrt{7}\right)\left(\sqrt{3}-\sqrt{7}\right)$

2. $\left(2\sqrt{a}-3\sqrt{b}\right)\left(2\sqrt{a}+3\sqrt{b}\right)$

3. $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$

10.3.2 Divide Radical Expressions

The Quotient Property of Radicals

If $a \geq 0$ and $b > 0$, then $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

Simplify.

1. $\sqrt{\frac{4x^6}{y^4}}$

2. $\sqrt{\frac{36a^4b^7}{3a^8b^2}}$

3. $\frac{\sqrt{9x^4y}}{\sqrt{xy}}$

4. $\frac{\sqrt{14u^6v}}{\sqrt{2u^2v^3}}$

Fact

A radical expression is not in simplest form if there is a radical in the denominator. To remove a radical from a denominator is to **rationalize tl denominator**. To rationalize a one-term denominator, multiply the numei and denominator of the fraction by the radical factor in the denominator aı simplify.

Simplify

1. $\frac{7}{\sqrt{10}}$

2. $\frac{-2}{\sqrt{5}}$

3. $\frac{-5}{\sqrt{ab}}$

4. $\frac{11}{2\sqrt{cd}}$

5. $\frac{2\sqrt{xy}}{x\sqrt{4y}}$

6, $\frac{3\sqrt{2ab}}{2a\sqrt{9b}}$

Rationalizing a Two-Term Denominator

When the denominator of a fraction is a square root expression with two terms, then rationalize the denominator by multiplying the numerator and denominator of the fraction by the conjugate of the denominator.

Simplify

1. $\frac{\sqrt{2y}}{\sqrt{y}+3}$

2. $\dfrac{-\sqrt{x}}{\sqrt{2x}-4}$

3. $\dfrac{\sqrt{x}+2}{2\sqrt{x}-3}$

4. $\dfrac{5-2\sqrt{3}}{4+3\sqrt{3}}$

5. $\dfrac{\sqrt{2}-2\sqrt{6}}{2\sqrt{2}-3\sqrt{6}}$

10.4 Solving Equations Containing Radicals

Property of Squaring Both Sides of an Equation

If $a = b$, then $a^2 = b^2$

Solve

1. $\sqrt{x} = 5$

2. $\sqrt{x-4} = 12$

Steps to Solve an Equation Containing One Square Root Term

1. Isolate the radical term.
2. Square both sides of the equation.
3. Solve the resulting equation.
4. Check your solution. Watch for extraneous solutions. **Extraneous solutions** are false solutions and do not satisfy the original equation.

Example 1

Solve $\sqrt{x-2} - 9 = 7$

1. Isolate the radical term.

2. Square both sides of the equation.

3. Solve the resulting equation.

4. Check your solution.

Solve

1. $\sqrt{4x} - 2 = 6$

2. $\sqrt{3x} + 5 = 4$

3. $\sqrt{3x} - 5 = 4$

4. $0 = 5 - \sqrt{2y - 3}$

5. $\sqrt{5x - 4} + 6 = 0$

Steps to Solve an Equation Containing Two Square Root Terms

1. Isolate one of the radical terms (preferably, the more complicated appearing term).
2. Square both sides of the equation.
3. Isolate the remaining radical term.
4. Square both sides of the equation.
5. Solve the resulting equation.
6. Check your solution(s). Watch for extraneous (false) solutions.

Solve

1. $\sqrt{9+x}-\sqrt{x}=6$

2. $\sqrt{x-20}+\sqrt{x}=10$

3. $\sqrt{9+2x}+\sqrt{2x}=6$

4. $\sqrt{5+3x}-\sqrt{3x}=9$