Exam 1 Sections and Topics

Chapter 1: Linear Functions

1.1 Describe Situations using Qualitative Graphs.
1.2 Sketching Graphs of Linear Functions
    i. Pick 3 *x*'s - Compute 3 *y*'s - Plot the 3 points
    ii. Find the *x*- and *y*-intercepts and graph.
    iii. Horizontal lines
    iv. Vertical lines
1.3 Slope of a Line
    i. Parallel lines
    ii. Perpendicular lines
1.4 Meaning of slope for Equations, Graphs and Tables
1.5 Finding Linear Equations
1.6 Relations and Functions

Chapter 2: Modeling with Linear Functions

2.1 Using Lines to Model Data
2.2 Finding Equations for Linear Models. Find the regression equations in all word problems with tables of data – do not do it the way shown in section 2.2.
2.3 Function Notation and Making Predictions
2.4 Slope as a Rate of Change

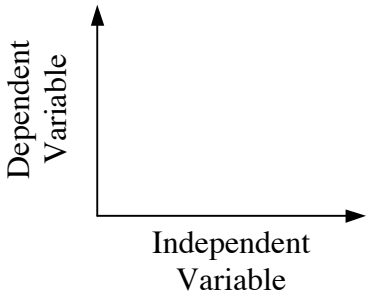
Chapter 3: Systems of Linear Equations

3.1 Using Graphs to Solve Systems of Equations
3.2 Using the Elimination and Substitution methods to Solve Systems of Equations
3.3 Using Systems to Model Data
3.4 Value, Interest and Mixture Problems
3.5 Linear Systems of Inequalities

Chapter 1 Linear Functions

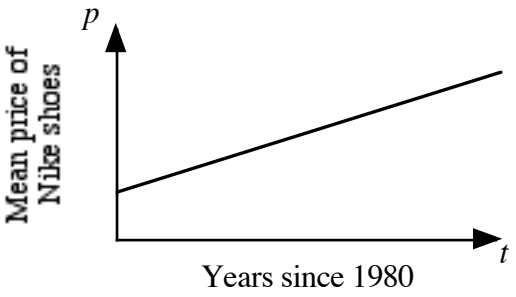
1.1 Using Qualitative Graphs to Describe Situations

A **qualitative graph** is a graph with no scale (i.e. tick marks and numbers) on the axes and it is used to describe the relationship between two variables. A graph or curve is said to be **linear** when it forms a straight line. When a variable y depends on another variable x, then we call y the **dependent variable** and x the **independent variable**. The independent variable is represented on the horizontal axis of a graph and the dependent variable is represented on the vertical axis.



Example 1

The retail price of Nike shoes has steadily increased since 1980. Let p be the mean price of Nike shoes and t represent the number of years since 1980. Thus, 1982 is represented by $t = 2$ years. The mean price of Nike shoes is described in the graph to the right.



a. Is the curve linear?

b. Does t depend on p or does p depend on t? That is, does the calendar year depend on the price of Nike shoes, or does the price of Nike shoes depend on the calendar year?

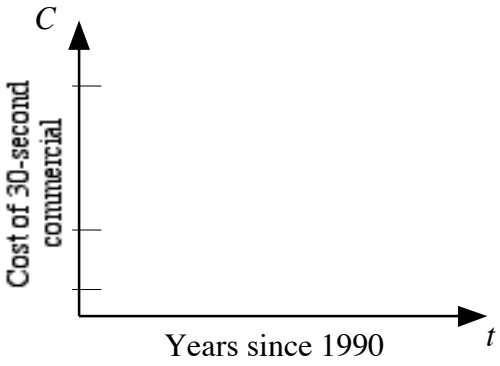
c. Which variable is the dependent variable?

d. Which variable is the independent variable?

e. What is the graph telling us?

Example 3

Let C represent the cost of a 30-second television commercial and t represent the number of years since 1990. Each year the cost of a commercial increased by more than the previous years increase (this type of curve is called an **exponential curve**). Sketch a qualitative graph that describes the relationship between C and t.



Example 4

Suppose hot coffee is poured into a cup at room temperature. Let F represent the temperature (in degrees Fahrenheit) of the coffee at t minutes since it was poured.

a. Which variable is the dependent variable? Why?

b. Which variable is represented along the horizontal axis?

c. Sketch a qualitative graph that describes the relationship between t and F. Explain any changes in the graph. As always, label the axes.

d. What does the F-intercept mean in this application?

Example 5

Suppose MegaDeath releases a new CD. Let n be the number of CDs sold when a dollars are spent on advertising.

a. Which is the dependent variable? Why? Which axis is it represented on?

b. If no money is spent on advertising, how many CDs will be sold? That is, is there a minimum number of CDs that will be sold?

c. Is there a maximum number of CDs that MegaDeath can sell? Why?

d. Sketch a qualitative curve that describes the relationship between a and n. Explain any changes in the graph. As always, label the axes.

Example 6

An airplane flew from San Diego to Los Angeles. Let a be the altitude of the plane t seconds after takeoff. Sketch a qualitative curve that describes the relationship between a and t. Explain any changes in the graph. As always, label the axes.

1.2 Sketching Graphs of Linear Equations

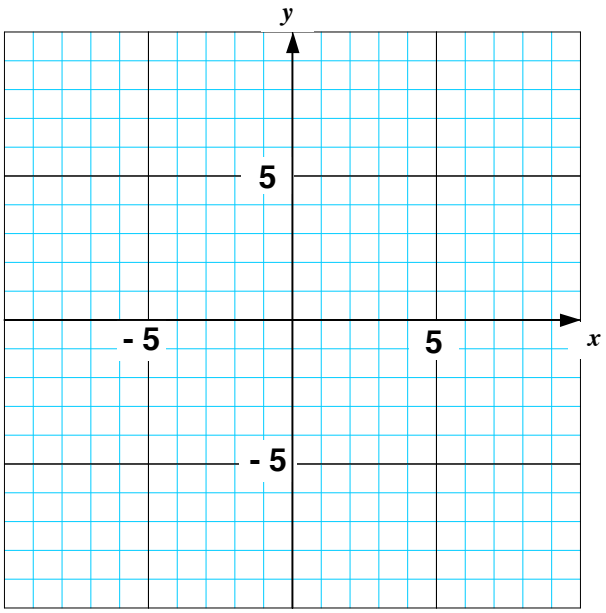
A number, or an ordered pair of numbers, is said to **satisfy an equation** if substituting the number(s) into the equation results in a true statement. An ordered pair (a, b) is a **solution of an equation in two variables** if the ordered pair satisfies the equation. The **solution set** of an equation is the set of all ordered pairs that satisfy the equation. The **graph of an equation** is a drawing of the solution set of the equation - it is formed by plotting all ordered pairs that satisfy the equation. An equation in the form $y = mx + b$, where m and b are constants and x and y are variables, is a **linear equation in two variables**. The graph of every linear equation in two variables ($y = mx + b$) forms a straight line.

Example 1 Consider the equation $y = -\frac{1}{4}x + 5$

a. Is the equation a linear equation in two variables?

b. Does (4, 4) satisfy the equation? Is (4, 4) a solution of the equation?

c. Find the ordered pair solutions of the equation corresponding to $x = -4$, and $x = 8$. Then graph the ordered pairs. Do the ordered pairs lie on the same straight line (called **collinear**)? If so, draw the line.



d. What is another ordered pair on the line? Does the ordered pair satisfy the equation?

Facts about Graphs of Equations

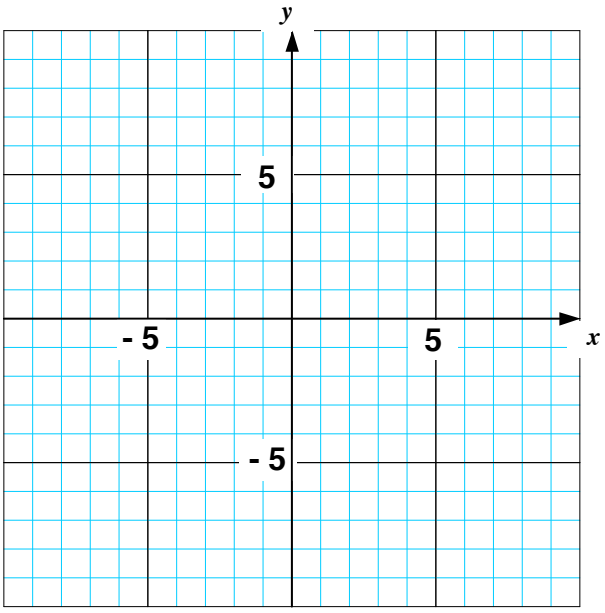
1. Every point on the graph of an equation satisfies the equation.
2. Every ordered pair that satisfies an equation is a point on the graph of the equation.
3. Points not on the graph of an equation do not satisfy the equation.

Steps to Sketch the Graph of a Line by Plotting Three Points

1. Solve the equation for y. That is, write the equation in the form $y = mx + b$.
2. Pick three values for the variable x.
3. Compute the three corresponding values for the variable y.
4. Plot the three ordered pairs and use a ruler to draw a line through the points. Extend the line through the entire grid.

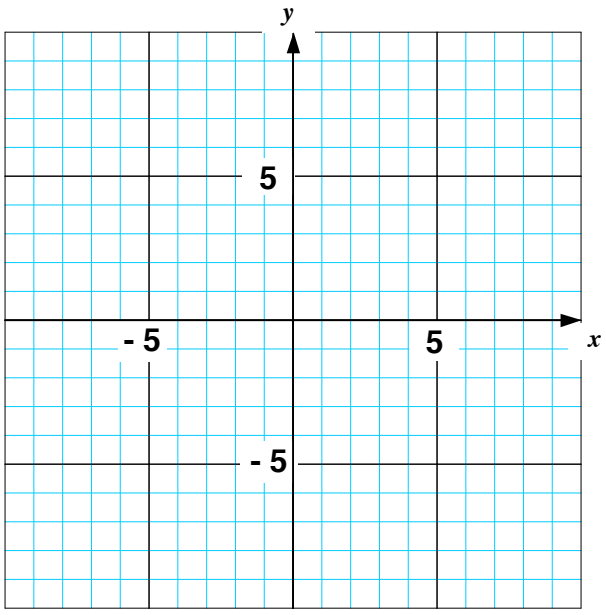
Example 2 Sketch the graph of $y = -2x + 5$.

Pick 3 values for x	**Compute** $y = -2x + 5$
$x = -1$	$y = -2(-1) + 5 =$
$x = 0$	$y = -2(\quad) + 5 =$
$x = 4$	$y = -2(\quad) + 5 =$



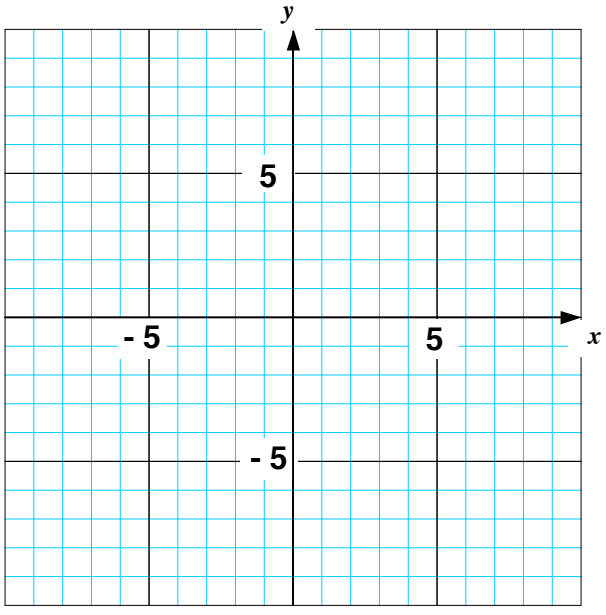
Example 3 Sketch the graph of $2x + 3y = 6$.

Pick 3 values for x	Compute $y =$



Example 4 Sketch the graph of $3(2y - 5) = 2x - 3 - 8x$.

Pick 3 values for x	Compute $y =$

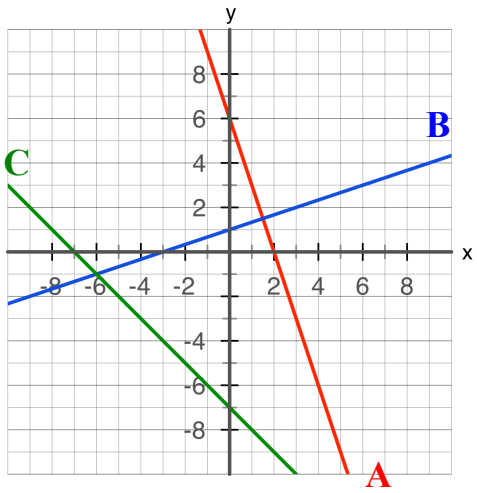


Finding the Intercepts of a Curve (Graph)

1. The ***x*-intercept** of a graph is the ordered pair location where the graph intersects the x-axis. An x-intercepts is written in the form $(a, 0)$, where a is a constant. To find the x-intercept, set $y = 0$ and solve for x.
2. The ***y*-intercept** of a graph is the ordered pair location where the graph intersects the y-axis. A y-intercepts is written in the form $(0, a)$, where a is a constant. To find the y-intercept, set $x = 0$ and solve for y.

Example 5

a. Identify the intercepts of each curve.



Curve	x-intercept	y-intercept
A	(,)	(,)
B	(,)	(,)
C	(,)	(,)

b. What is the y value of all x-intercepts?

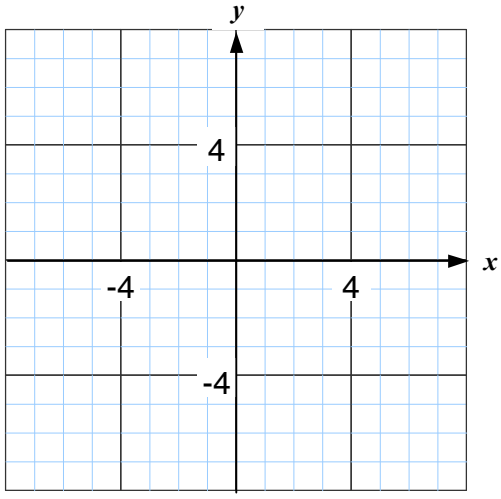
c. What is the x value of all y-intercepts?

Example 6

Find the intercepts of $2y = -x + 6$. Then sketch its graph.

x-intercept: _____

y-intercept: _____



Example 7 Graph by Finding the Intercepts

a. Find the intercepts and sketch the graph of $-4y+8x=24$.

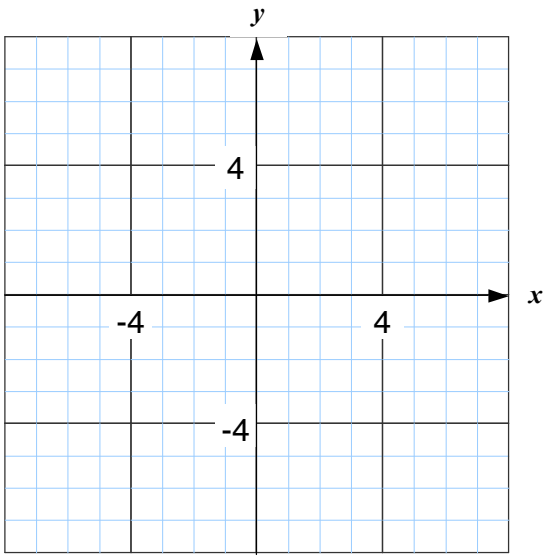
x-intercept: _____

y-intercept: _____

b. Find the intercepts and sketch the graph of $6x-9y=18$.

x-intercept: _____

y-intercept: _____



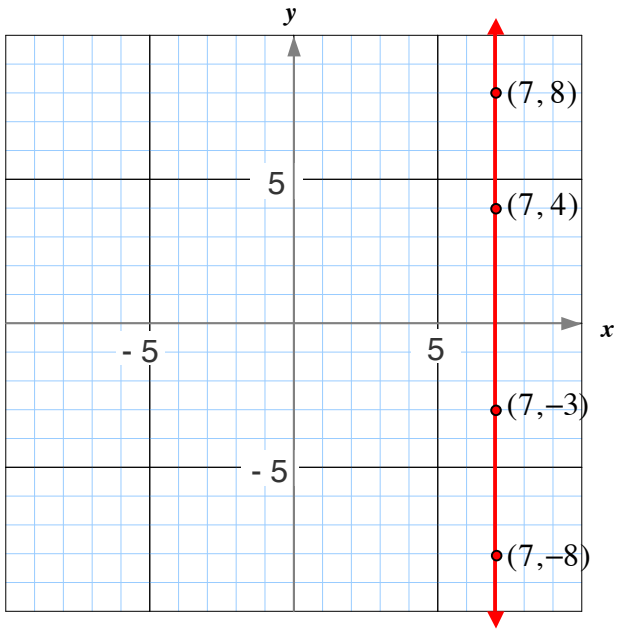
Example 8 Graph and Equation of a Vertical Line

The graph of the line containing the points $(7,-8)$, $(7,-3)$, $(7,4)$ and $(7,8)$ is shown.

a. What kind of line is formed (horizontal or vertical)?

b. What is common to every ordered-pair (x, y) on the line?

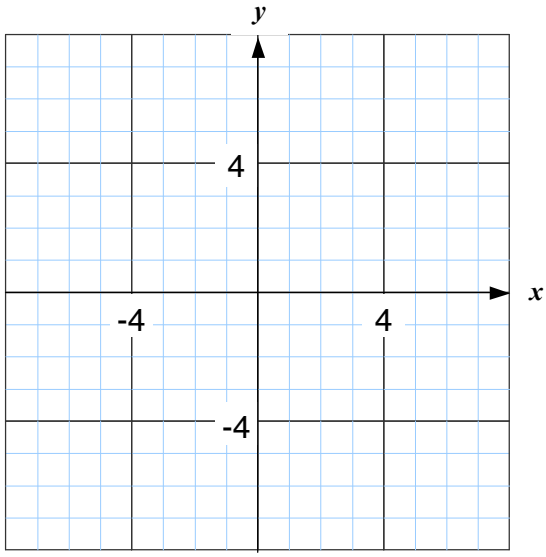
c. What is the equation of the line?



d. Write the *x*-intercept: _____ and *y*-intercept: _____

Example 9 Graph and Equation of a Horizontal Line

a. Sketch the graph of the line containing the points $(4,-3)$, $(-6,-3)$, $(0,-3)$ and $(-1,-3)$.



b. What kind of line is formed?

c. What is common to every point (x, y) on the line?

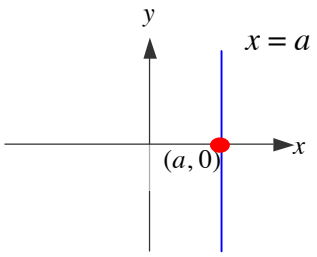
d. What is the equation of the line?

e. What is the x-intercept: _____

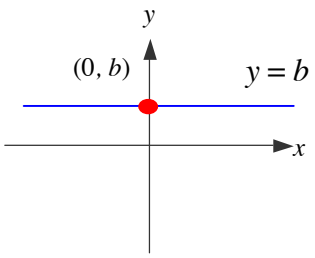
What is the y-intercept: _____

Equations of Horizontal and Vertical Lines

1. All vertical lines have an equation in the form $x = a$, where a is a constant. The x-intercept is $(a, 0)$ and there is no y-intercept.



2. All horizontal lines have an equation in the form $y = b$, where b is a constant. The y-intercept is $(0, b)$ and there is no x-intercept.



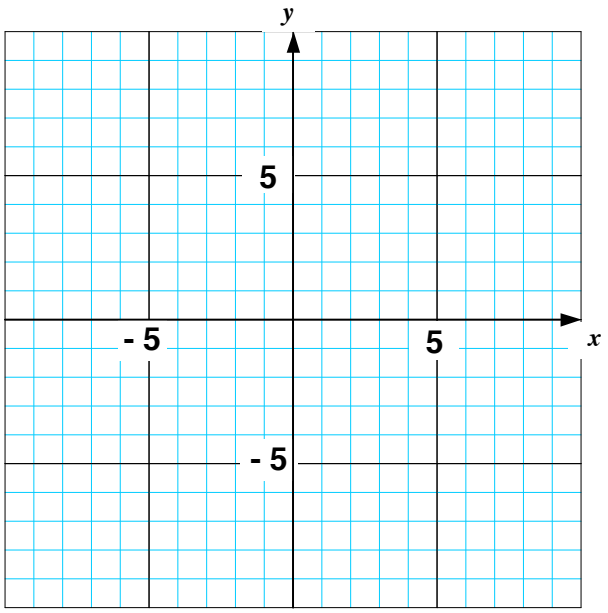
Example 9

Sketch each graph and write the intercepts.

a. $y = 7$

x-intercept _____

y-intercept _____



b. $x = -3$

x-intercept _____

y-intercept _____

c. Sketch the line $x = 0$. What is another name for it?

d. Sketch the line $y = 0$. What is another name for it?

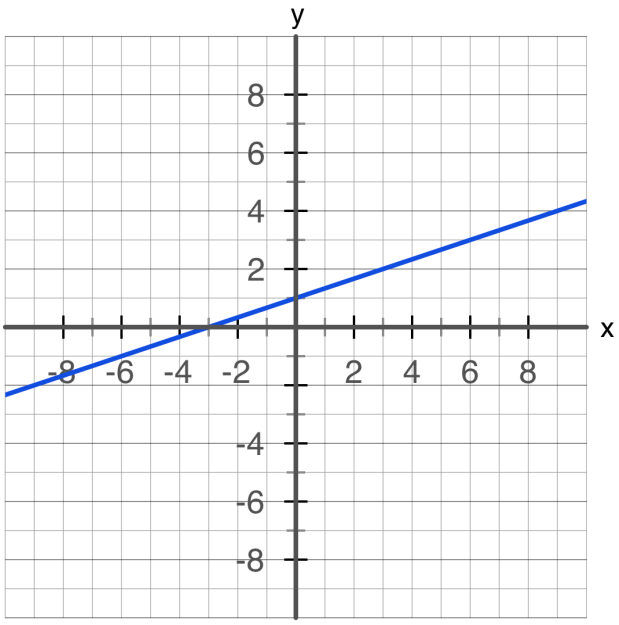
Example 11 Reading *x*- and *y*-values from Graphs

a. Estimate y when $x = 4$.

b. Estimate y when $x = -6$.

c. Estimate x when $y = 2$.

d. Estimate x when $y = 0$.



1.3 Slope of a Line

The measure of a line's slant is called the **slope** of the line, denoted m. Vertical lines have no slope, or the slope is said to be undefined. The slope of a non-vertical line containing the two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

read "change in y"

read "change in x"

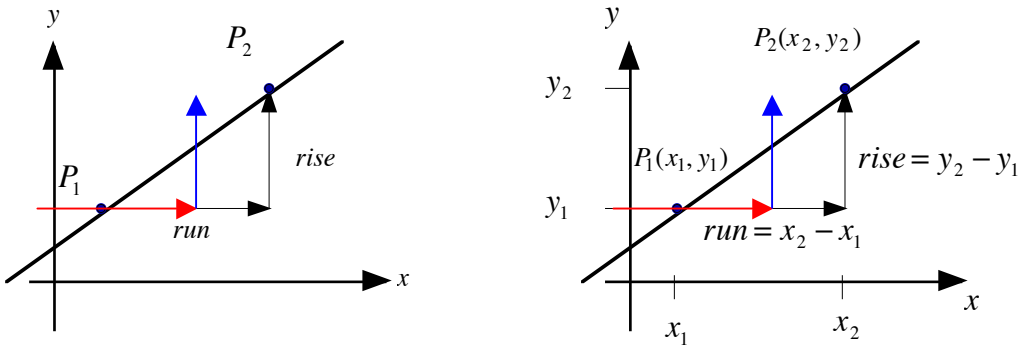
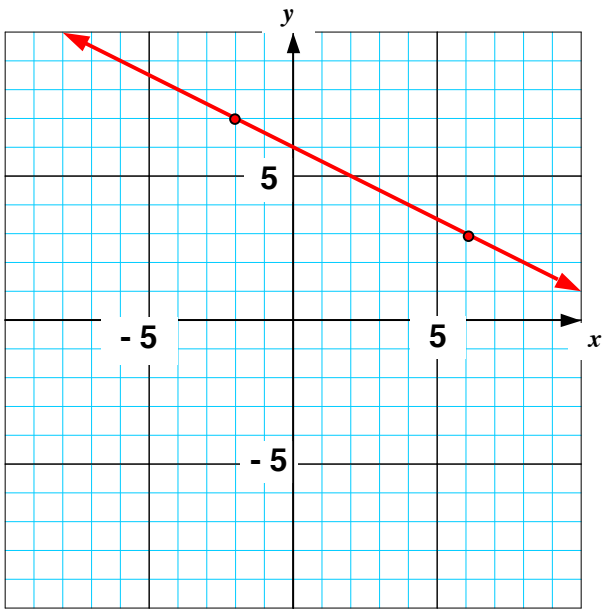


Figure 1 Slope of a line, $m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$

Example 1

a. The line containing the points $P_1(-2,7)$ and $P_2(6,3)$ is shown. Compute the slope of the line.

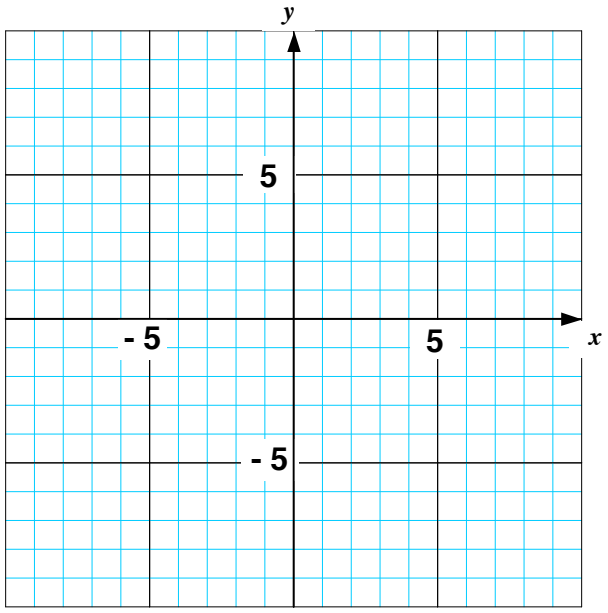


b. Compute its slope using two other points on the line.

c. Decreasing lines have a [positive negative] slope. (circle one)

Example 2

a. Graph the line containing the points $P_1(-3,-5)$ and $P_2(5,1)$. Then compute the slope of the line.

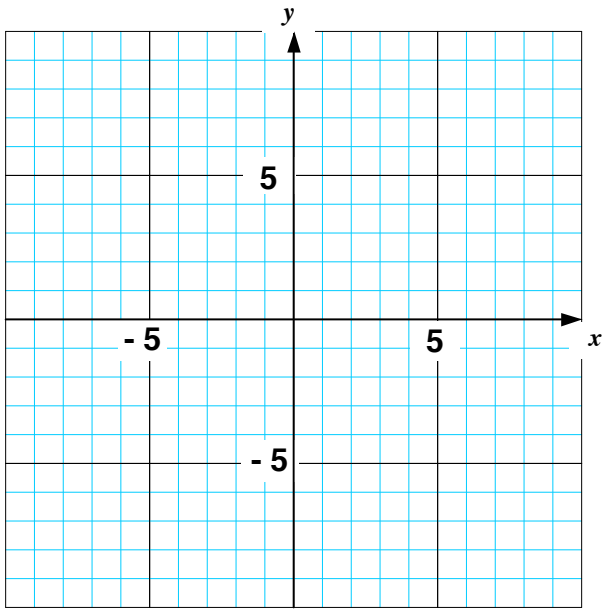


b. Increasing lines have a [positive negative] slope. (circle one)

Example 3

Graph the line containing the given points. Then compute the slope of the line.

a. (6,4) and (6,1)



b. (–7,–5) and (1,–5)

c. Horizontal lines have a [positive negative zero undefined] slope.

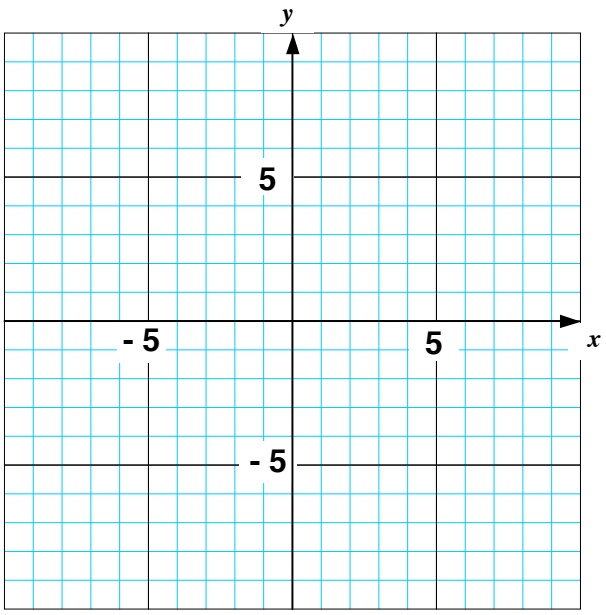
d. Vertical lines have a [positive negative zero undefined] slope.

e. List three more ordered pairs on the line in part **a**. What is the equation of the line?

f. List three more ordered pairs on the line in part **b**. What is the equation of the line?

Example 4

a. Graph the line containing the points $(6,3)$ and $(-2,-7)$. Then compute the slope of the line.



b. Graph the line containing the points $(4,3)$ and $(-4,-7)$. Then compute the slope of the line.

c. What can be said about the two lines from parts **a** and **b**?

Slopes of Parallel lines

Two distinct non-vertical lines are parallel if and only if their slopes are equal. All distinct vertical lines are parallel.

Slopes of Perpendicular lines

Two lines are perpendicular if and only if their slopes are opposite reciprocals of each other. Any horizontal line is perpendicular to any vertical line.

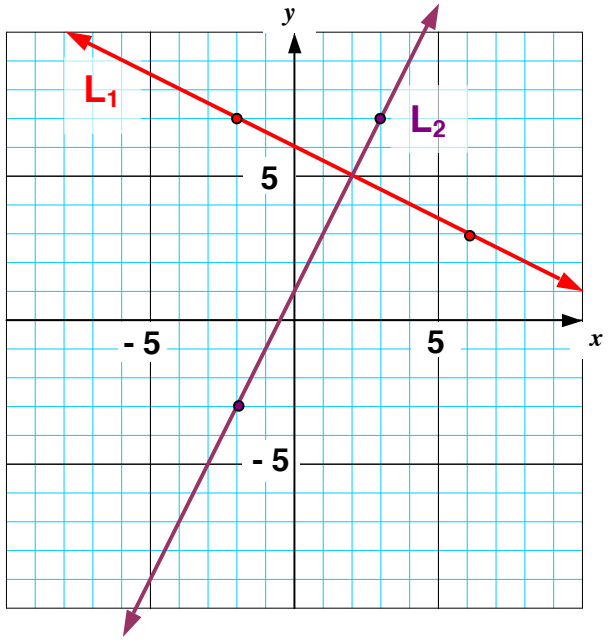
Example 5

The slope of line l is given. For each find the slope of the line parallel to line l and perpendicular to line l.

Slope of line l	Slope of any line parallel to l	Slope of any line perpendicular to l
$\frac{2}{3}$		
-4		
$-\frac{3}{8}$		
0		
undefined		

Example 6

The line L_1 contains the points $(-2,7)$ and $(6,3)$, and the line L_2 contains the points $(-2,-3)$ and $(3,7)$ are shown. Compute the slope of each line. What can be said about the two lines?



Example 6

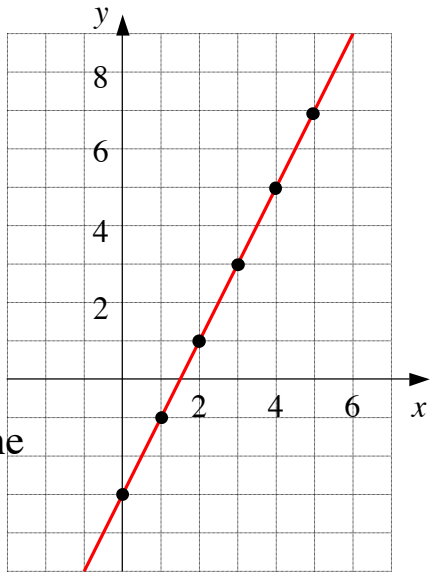
Road A climbs steadily for 135 feet over a horizontal distance of 3900 feet.
Road B climbs steadily for 120 feet over a horizontal distance of 3175 feet.
Which road is steeper? Explain.

1.4 Meaning of Slope for Equations, Graphs and Tables

Example 1 Consider the equation $y = 2x - 3$ and its graph.

a. Compute the slope of the line?

x	*y*
0	-3
1	-1
2	1
3	3
4	5
5	7



What is the coefficient of x in the equation $y = 2x - 3$?

Each time x increases by one unit, what happens to the value of y?

If the run is one, what is the rise?

How are the above values related?

b. What is the y-intercept in the graph?

What is the constant term in the equation $y = 2x - 3$?

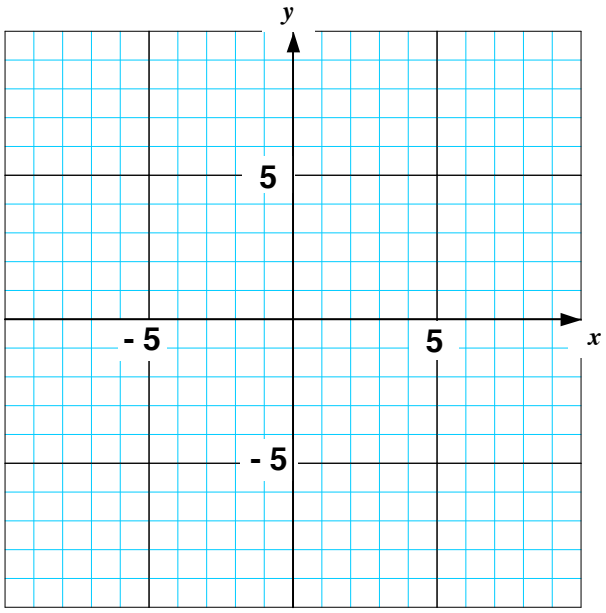
What is the relationship of the two values?

Slope-Intercept form a Line, $y = mx + b$

The **slope-intercept form of a line** is $y = mx + b$, where the slope of the line is $\boldsymbol{m}$, and the y-intercept is located at the point $(0, \boldsymbol{b})$.

Example 2

Sketch the graph of $y = -3x + 5$ using the slope and y-intercept.

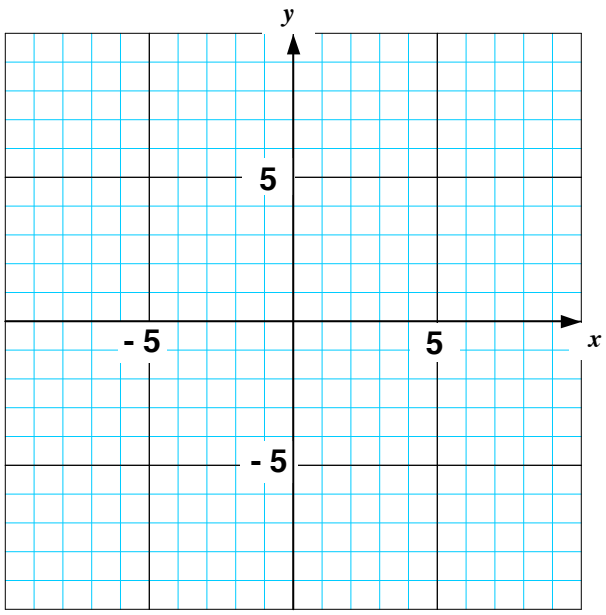


Example 3

Sketch the graph of $y = \frac{1}{3}x - 2$ using the slope and y-intercept.

Example 4

Sketch the graph of $-3x + 5y = 12$ using the slope and y-intercept.



Slope Addition Property

For any linear equation, if the value of the independent variable is increased by 1, then the value of the dependent variable changes by the slope m (increasing if m is positive and decreasing if m is negative).

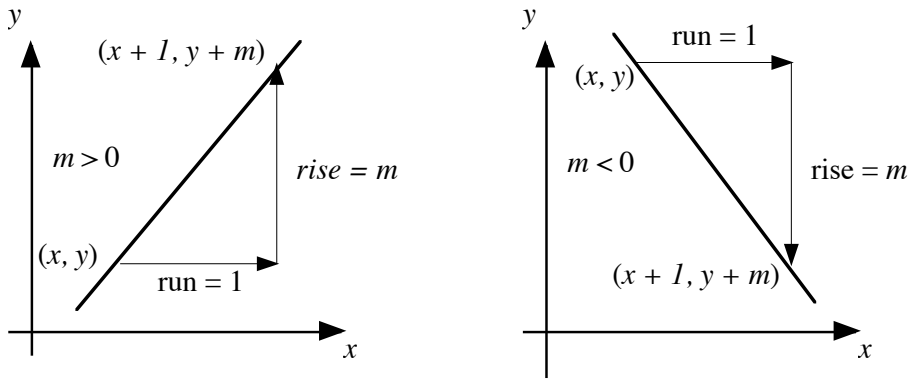


Figure 1 Whether the slope is positive or negative, when the run = 1, the slope = rise = m

Example 5

Five sets of points are described in the following table. For each set, decide if there is a line that passes through every point. If so, find the slope of that line. If not, decide whether there is a line that comes close the every point.

Set 1		Set 2		Set 3		Set 4		Set 5	
x	y	x	y	x	y	x	y	x	y
1	50	1	6	0	8	1	21	7	-3
2	47	2	106	1	8	12	27	7	4
3	44	3	205	2	8	32	33	7	17
4	41	4	305	3	8	45	39	7	50
5	38	5	406	4	8	57	45	7	25
6	35	6	505	5	8	64	51	7	99

1.5 Finding Linear Equations

Slope-Intercept form a Line, $y = mx + b$

The slope-intercept form of a line is $y = mx + b$, where the slope of the line is m, and the y-intercept is located at the point $(0, b)$.

Example 1 Find the equation of a line given a point on the line and its slope

Find the equation of the line that has slope $m = -2$ and contains the point (-4, 5).

Step 1 The goal is to determine the values for m and b in $y = mx + b$. Since we are given the slope, we know $m = -2$ so that the equation, so far, becomes $y = -2x + b$

Step 2 Use the point (-4, 5) and the equation $y = -2x + b$ to determine the constant b.

Step 3 Write the equation in the form $y = mx + b$.

Example 2 Find the equation of a line given a point on the line and a parallel line

Find the equation of the line that contains the point (-2, 7) and is parallel to the line $y = \frac{3}{5}x - 2$.

Example 3 Find the equation of a line given a point on the line and a perpendicular line

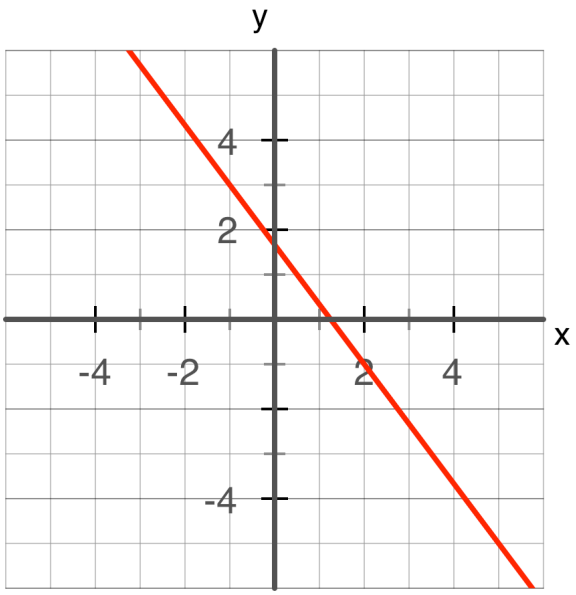
Find the equation of the line that contains the point (5, -4) and is perpendicular to the line $4x - 5y = -15$.

Example 4 Find the equation of a line given two points on the line

Find the equation of the line that contains the points (-4, -1) and (3, 5).

Example 5 Find the equation of a line given its graph

Find the equation of the line shown.



Example 6 Find the equation from a table of values

Let y represent a person's salary (in thousands of dollars) after working at a company for x years.

a. Find the equation that describes the relationship between x and y.

Time at Company (years) x	Salary ($1000) y
0	25
1	28
2	31
3	34
4	37
5	40

b. Explain the meaning of the slope in this situation.

1.6 Relations and Functions

Example 1

Suppose seven purchases were made at GasMart stores around the country (see table). The relationship between the two quantities is called a **relation**. The set of all input values (gallons purchased) is called the **domain** of the relation. The set of all output values (total cost) is called the **range** of the relation.

Number of Gallons Purchased *x*	Total Cost (dollars) *y*
6.2	$10.45
13.0	$21.19
24.1	$45.60
8.8	$16.56
10.4	$17.38
9.6	$17.38
13.0	$20.64
Domain member	Range member
Input value	Output value
Independent variable	Dependent Variable

Relation, Domain and Range

A **relation** is any set of ordered pairs. The set of all **independent variable** values (**input** values) forms the **domain** of the relation. The set of all **dependent variable** values (**output** values) forms the **range** of the relation. Relations can be described in tables, as sets of ordered pairs, by diagrams, by equations, and by graphs.

Example 2

a. Express the relation in example 1 as a set of ordered pairs.
{(6.2, 10.45), (13.0, 21.19), (24.1, 45.60), (8.8, 16.56), (10.4, 17.38), (9.6, 17.38), (13.0, 20.64)}

b. What is the domain of the relation?

c. What is the range of the relation?

d. Identify the independent variable: Gallons or Cost
Identify the dependent variable: Gallons or Cost

Example 3

The relation of example 1 expressed as a output diagram is illustrated in figure 1.

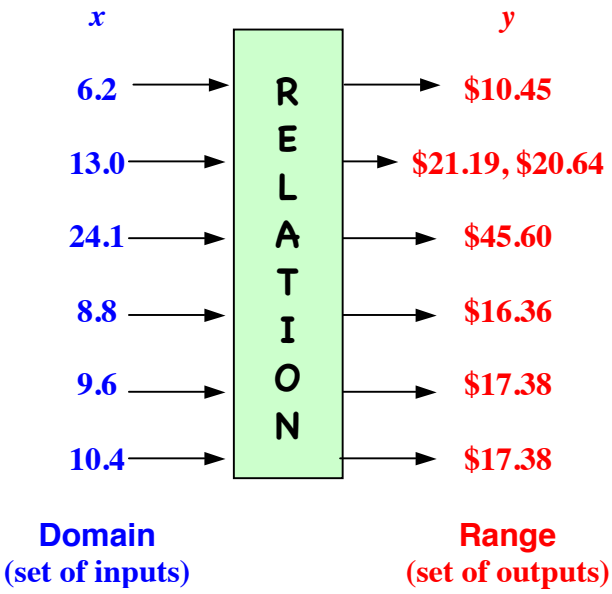


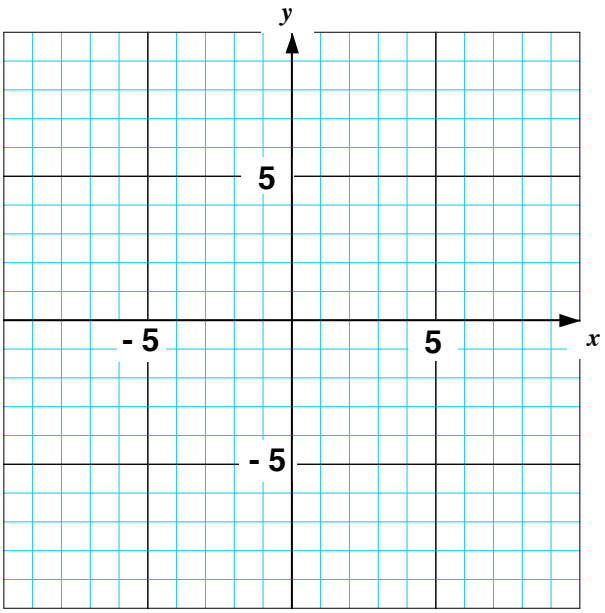
Figure 1 A relation as an input-output diagram

Example 4

a. Express the relation $y = -\frac{2}{3}x + 4$ as a graph.

b. State the domain and range of the relation.

c. Does each input yield exactly one output?



Example 4 is a special type of relation called a **function**.

Definition of a Function

A **function** is a relation if each **input** gives *exactly one* **output**. The set of all **independent variable** values (**input** values) forms the **domain** of the function. The set of all **dependent variable** values (**output** values) forms the **range** of the function.

Example 5

a. Is the relation shown also a functi
Explain.

b. What point(s) can be removed fro
relation so that a function is forme

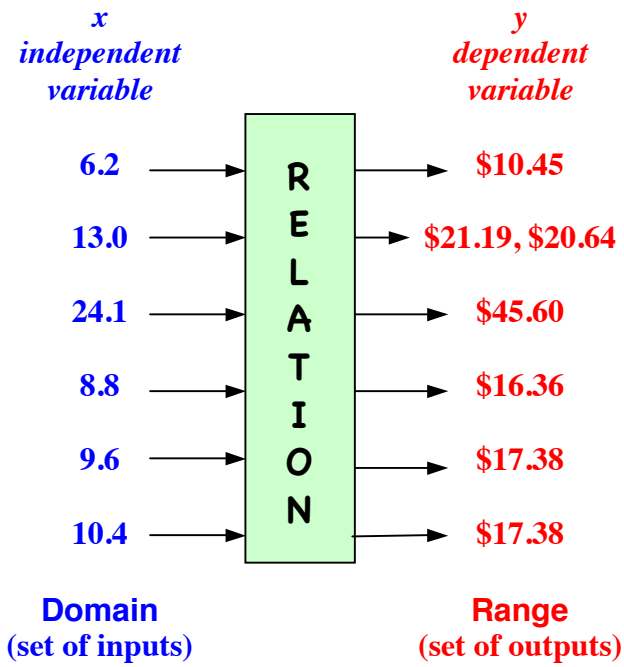


Figure 1 A relation as an input-output diagram

Example 6

a. Is the relation $y = \pm x$ a function? Explain.

b. Is $y = -\frac{2}{3}x + 4$ a function? Explain.

Example 7

Is the relation $x^2 = y$ a function? Explain.

Example 8

Is the relation $y^2 = x$ a function? Explain.

Example 9

Is the relation in figure 2 a function? Explain.

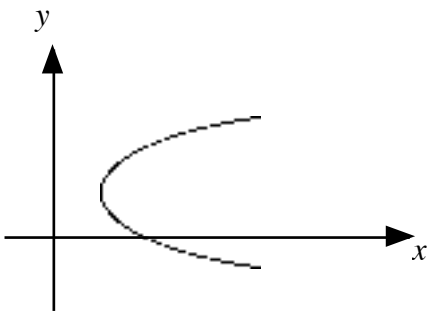


Figure 2 Is this relation a function?

Vertical Line Test

A graph is a graph of a function if every vertical line intersects the graph at most once (i.e. one time or not at all).

Example 10

Which of the following curves are also functions? Explain.

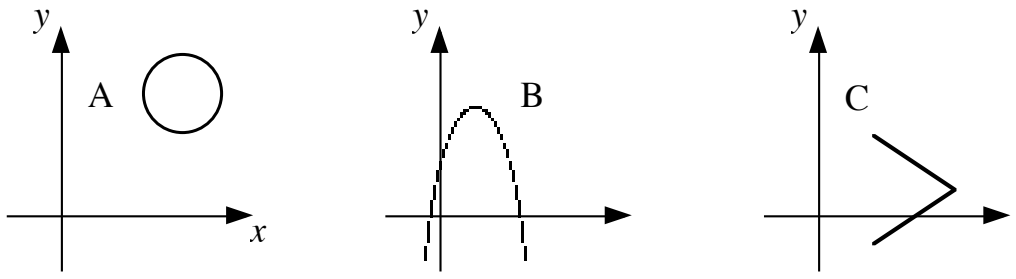
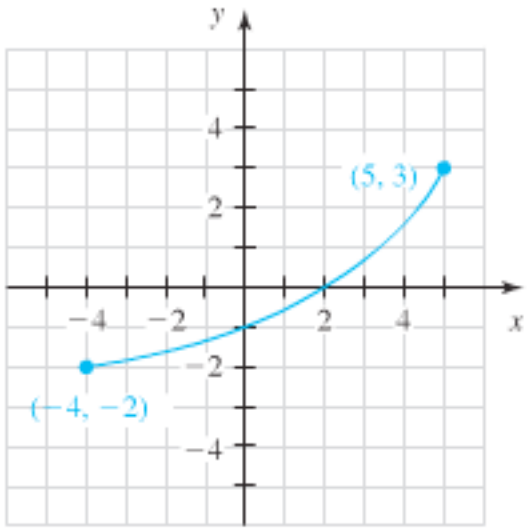


Figure 3 Which curves are graphs of functions?

Example 11

Determine the domain and range of each function.

a.



b.